

6.1 differential equations

equation that contains a derivative

ex. $\frac{dy}{dx} = \sec^2 x + 6x^2 + 5$

solve the differential equation

find y
 specific solutions $\left\{ \begin{array}{l} y = \tan x + 2x^3 + 5x \\ y = \tan x + 2x^3 + 5x + 6 \end{array} \right.$

general solution $\rightarrow y = \tan x + 2x^3 + 5x + C$

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initial value problem

a differential equation with initial conditions

ex: $\frac{dy}{dx} = e^x - 2x$ $(1, 3)$ $\begin{matrix} \uparrow \\ \text{point on } y = F(x) \\ X=1 \\ Y=3 \end{matrix}$

1. find general solution.

$$y = e^x - x^2 + C$$

2. use initial conditions to find C

(plug in) $3 = e - 1 + C$
 $C = 3 + 1 - e$

$C = 4 - e$
 $y = e^x - x^2 + 4 - e$

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$$\frac{dy}{dx} = e^{-x^2} \quad (7, 3) \quad \{\text{initial conditions}\}$$

$$y = \int_a^x e^{-t^2} dt \quad \text{describes a function}$$

$$\frac{dy}{dx} = \frac{d}{dx} \int_a^x e^{-t^2} dt = e^{-x^2}$$

$$f_1(x) = \int_0^x e^{-t^2} dt$$

$$y = \int_7^x e^{-t^2} dt + 3$$

same

$$f_2(x) = \frac{d}{dx} f_1(x)$$

$$f_2(x) = e^{-x^2}$$

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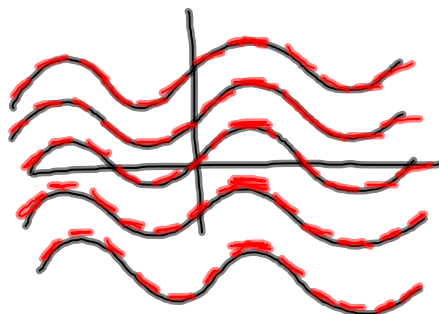
visual solution

$$\frac{dy}{dx} = \cos x$$

solve:

$$y = \sin x + c$$

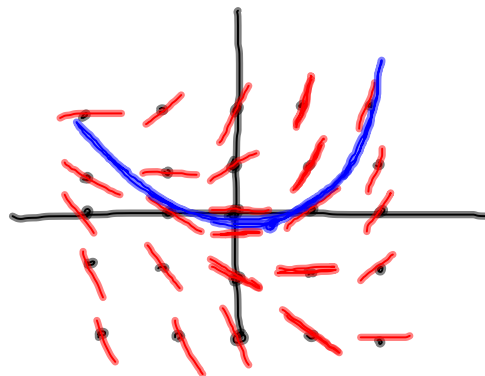
slope field

family
of
solutions

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build your own slopefield

$$\frac{dy}{dx} = x + y$$



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